Equivalência entre a Área sob a Curva Kolmogorov-Smirnov e o Índice de Gini na Avaliação de Desempenho de Decisões Binárias

Paulo ADEODATO

Sílvio Melo

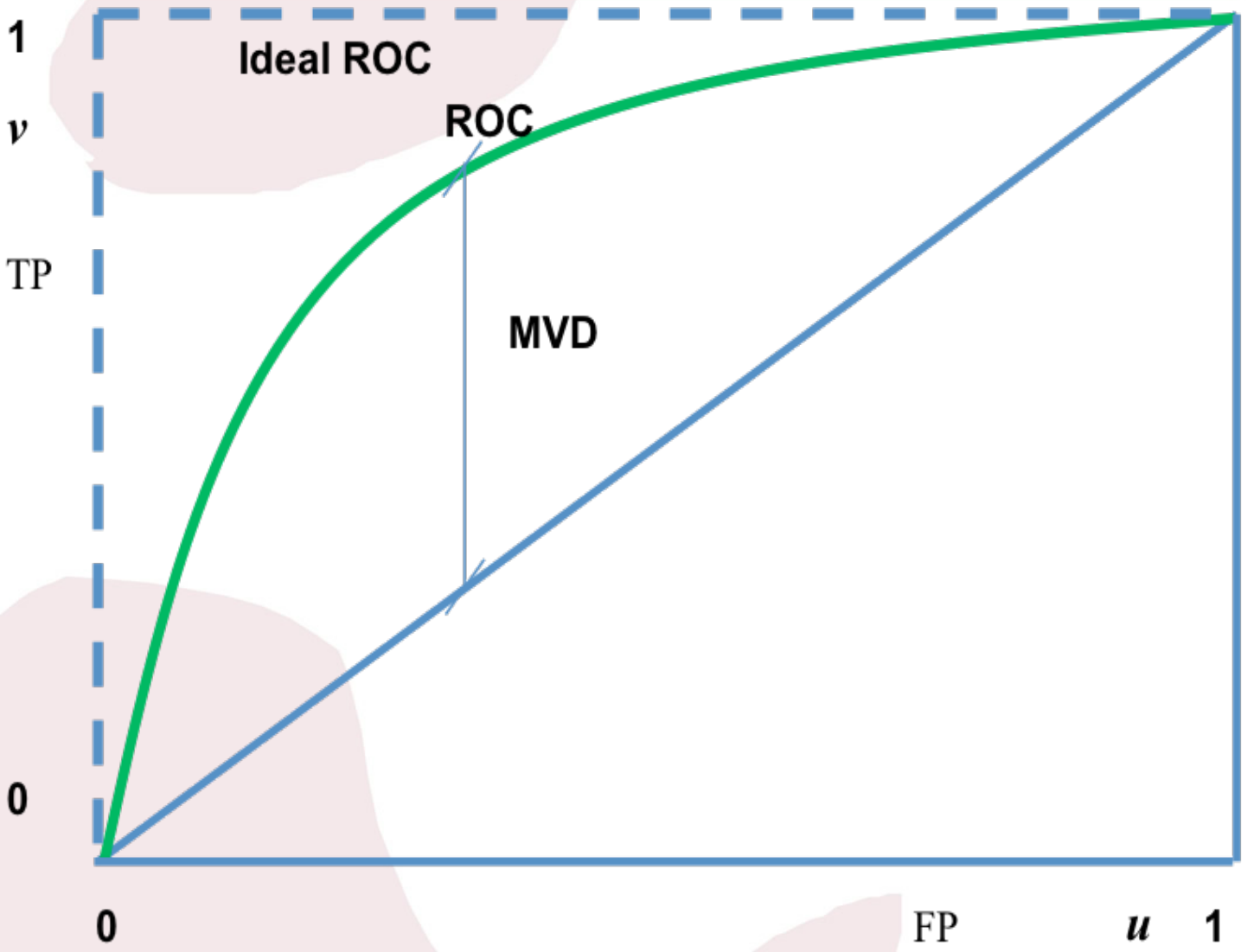
Roteiro

1. Decisão binária
2. Curva ROC (Receiver Operating Characteristics)
3. Curva KS (Kolmogorov-Smirnov)
4. Curva de Lorenz e Índice de Gini
5. Equivalência ROC x KS
6. Equivalência Gini x KS
7. Classificadores reais
8. Conclusões

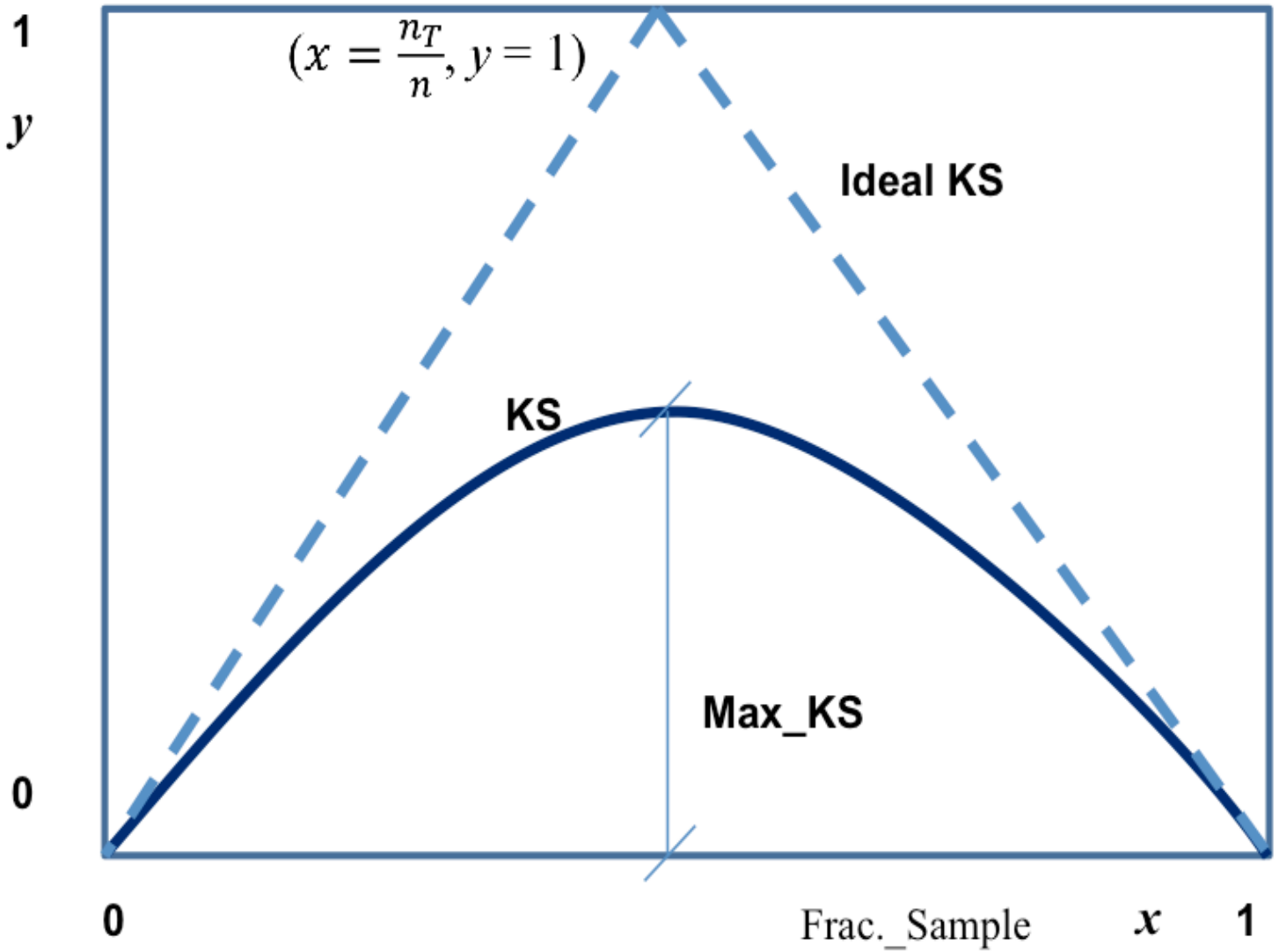
Decisão Binária

- Primeira decisão, mais importante e mais frequente
- Classificadores de resposta contínua (Rankers)
- Decisão por limiar: Controle do ponto de operação, simulação
- Métricas pontuais: Taxa de erro, acurária, f-measure etc.
- Métricas baseadas em área: AUC_ROC, Índice de Gini

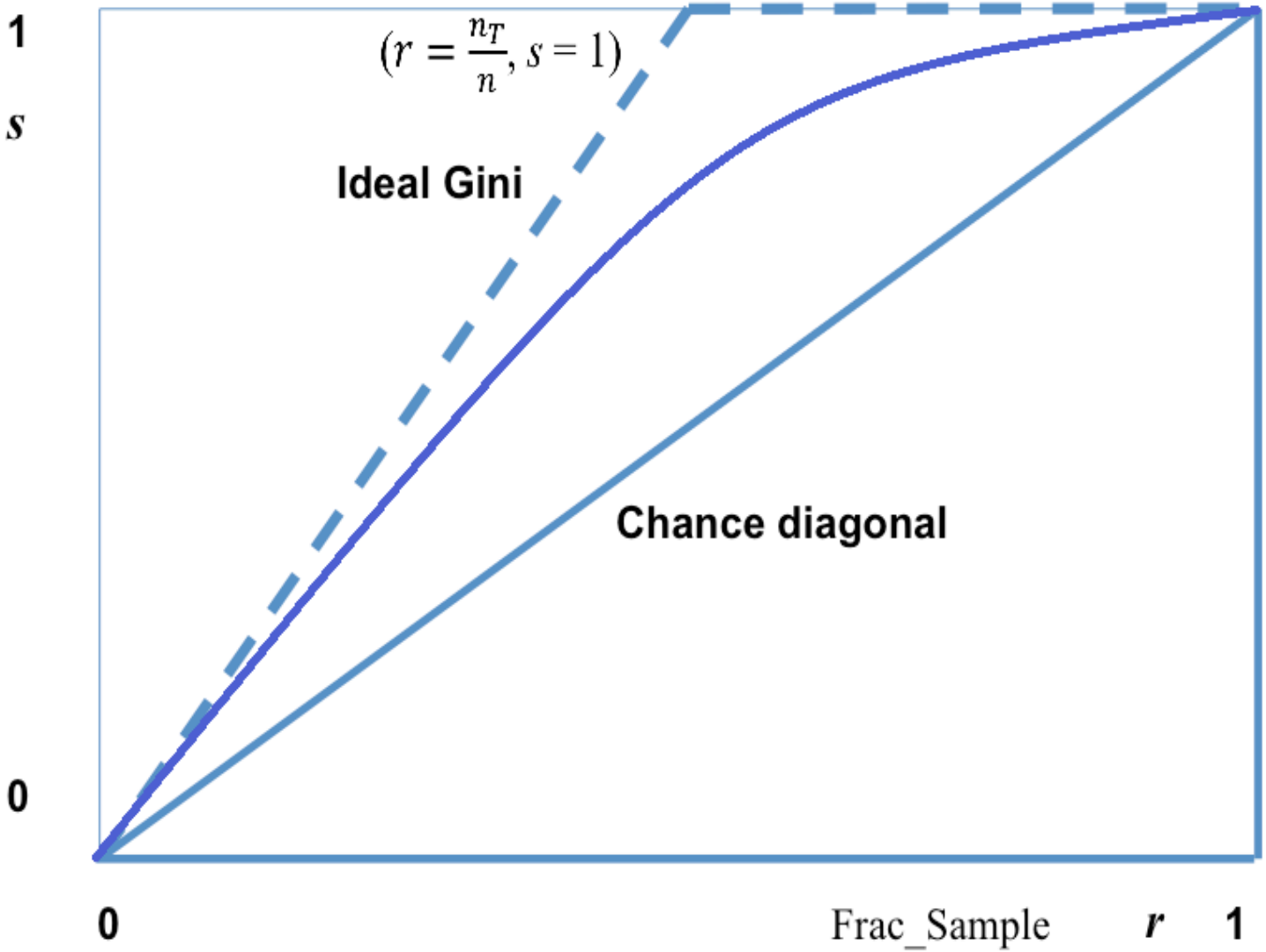
Curva ROC



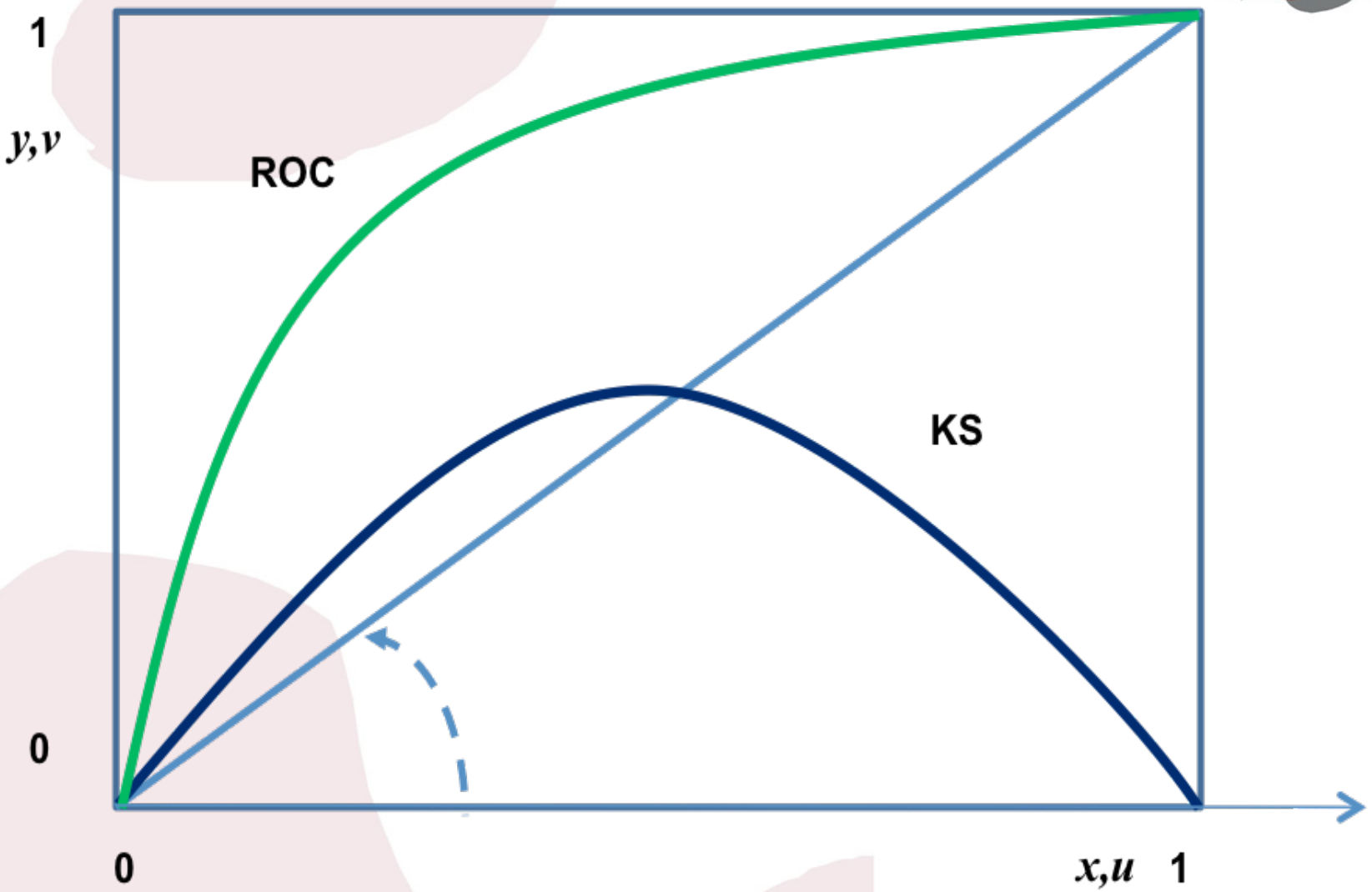
Curva KS



Curva de Lorenz



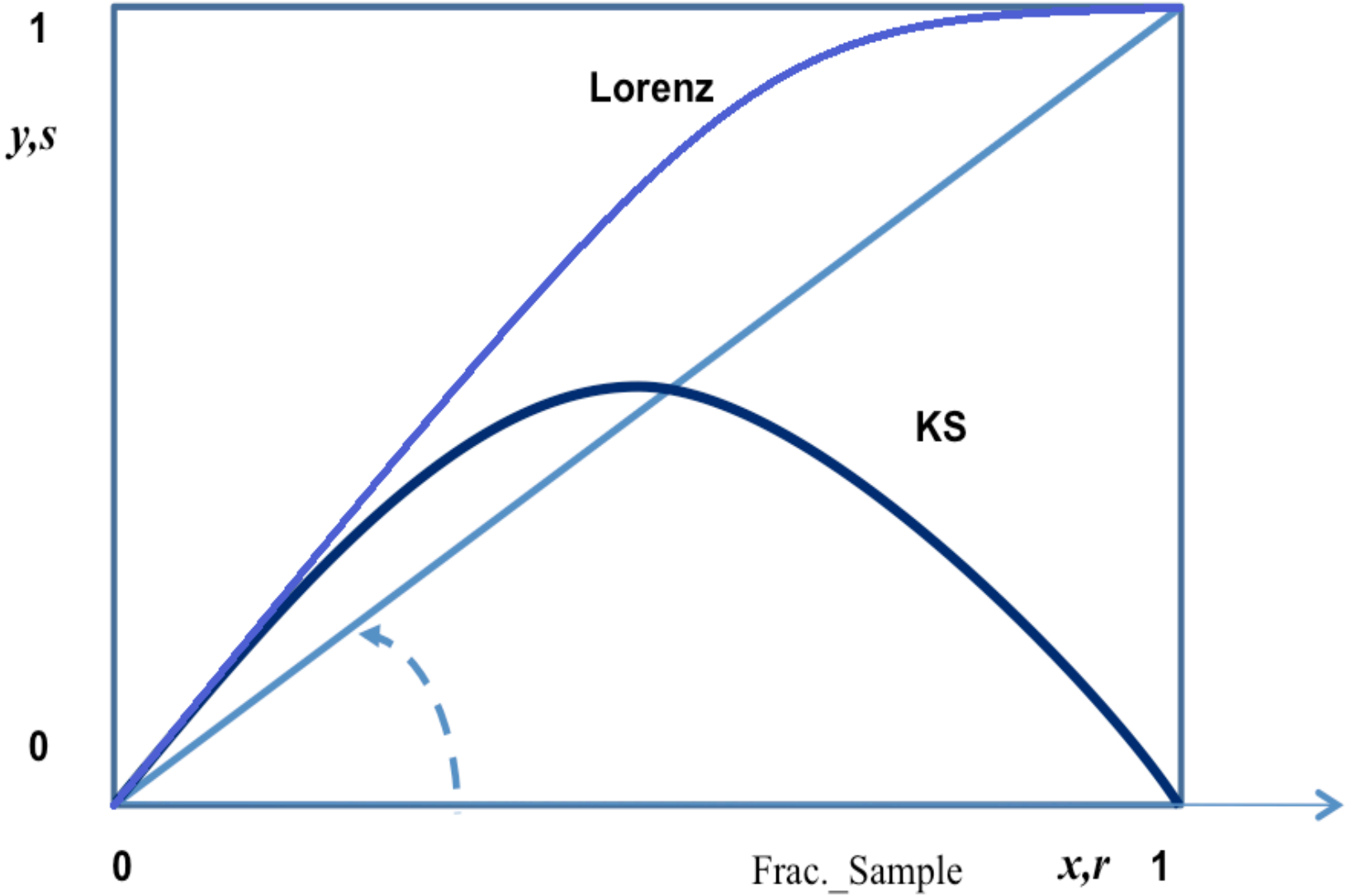
Equivalência ROC x KS



Equivalência ROC x KS

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & -\frac{n_T}{n} \\ 1 & \left(1 - \frac{n_T}{n}\right) \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

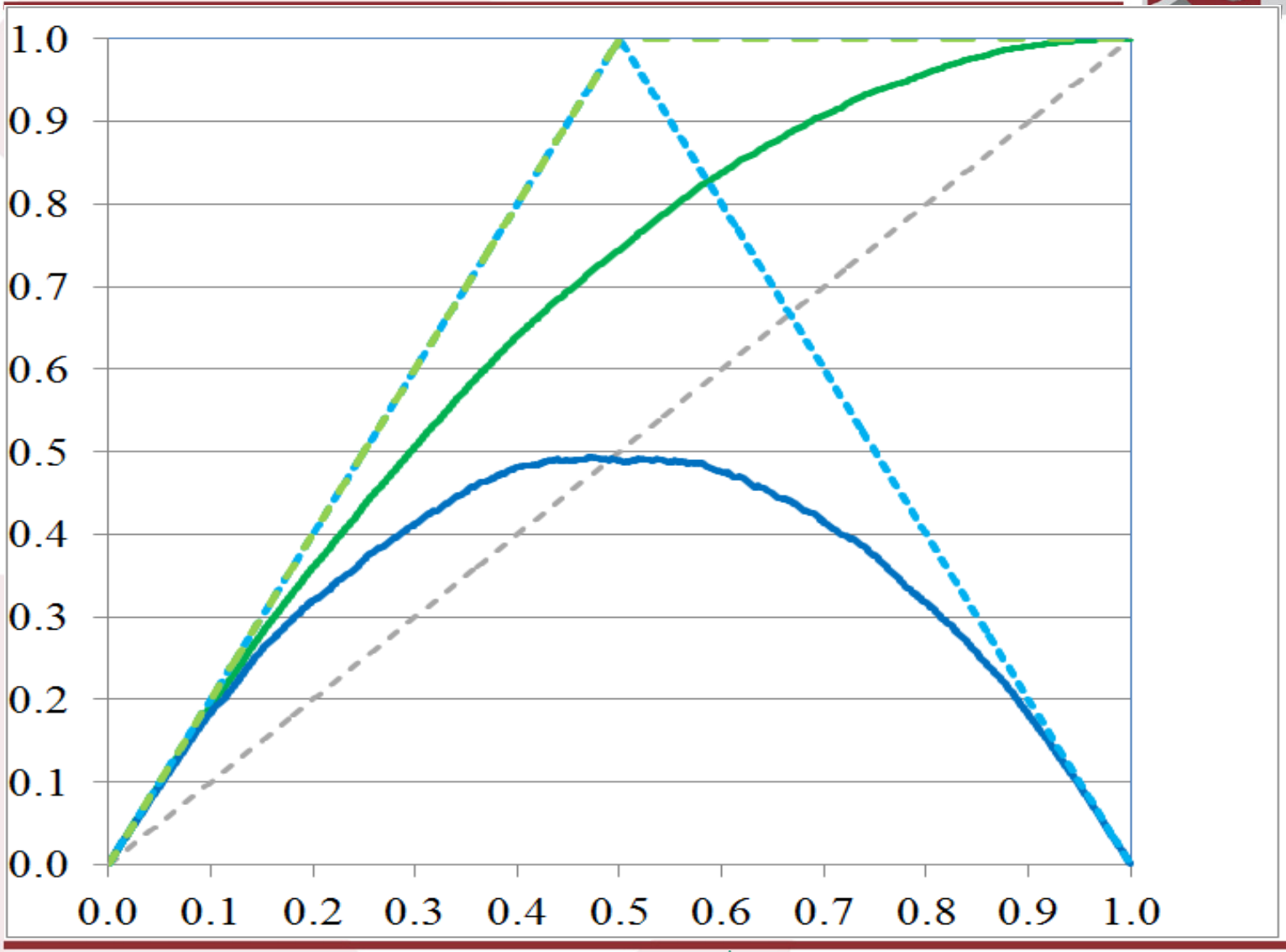
Equivalência Lorenz x KS



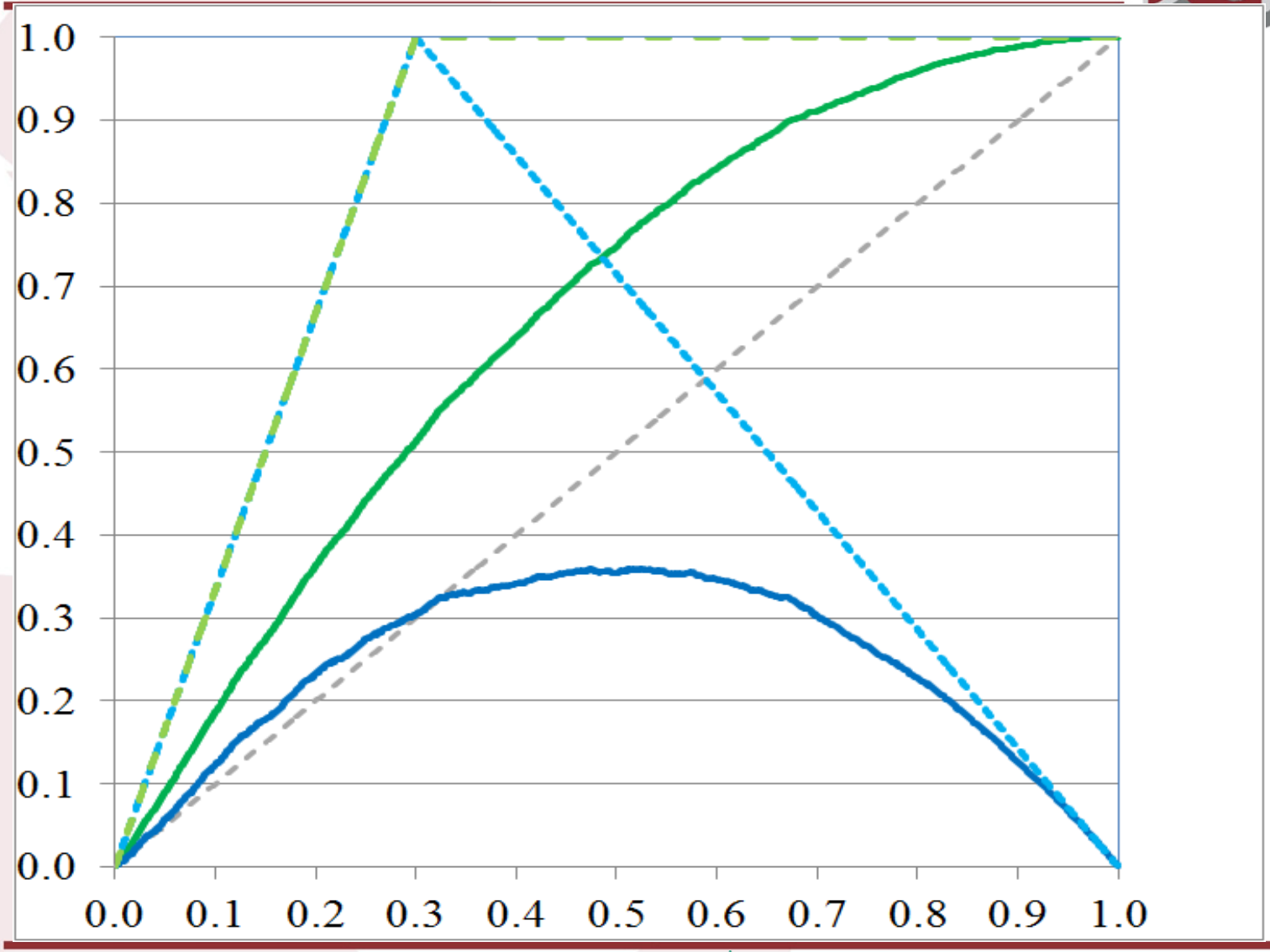
Equivalência Lorenz x KS

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & \left(1 - \frac{n_T}{n}\right) \end{bmatrix} \cdot \begin{bmatrix} x \\ y \end{bmatrix}$$

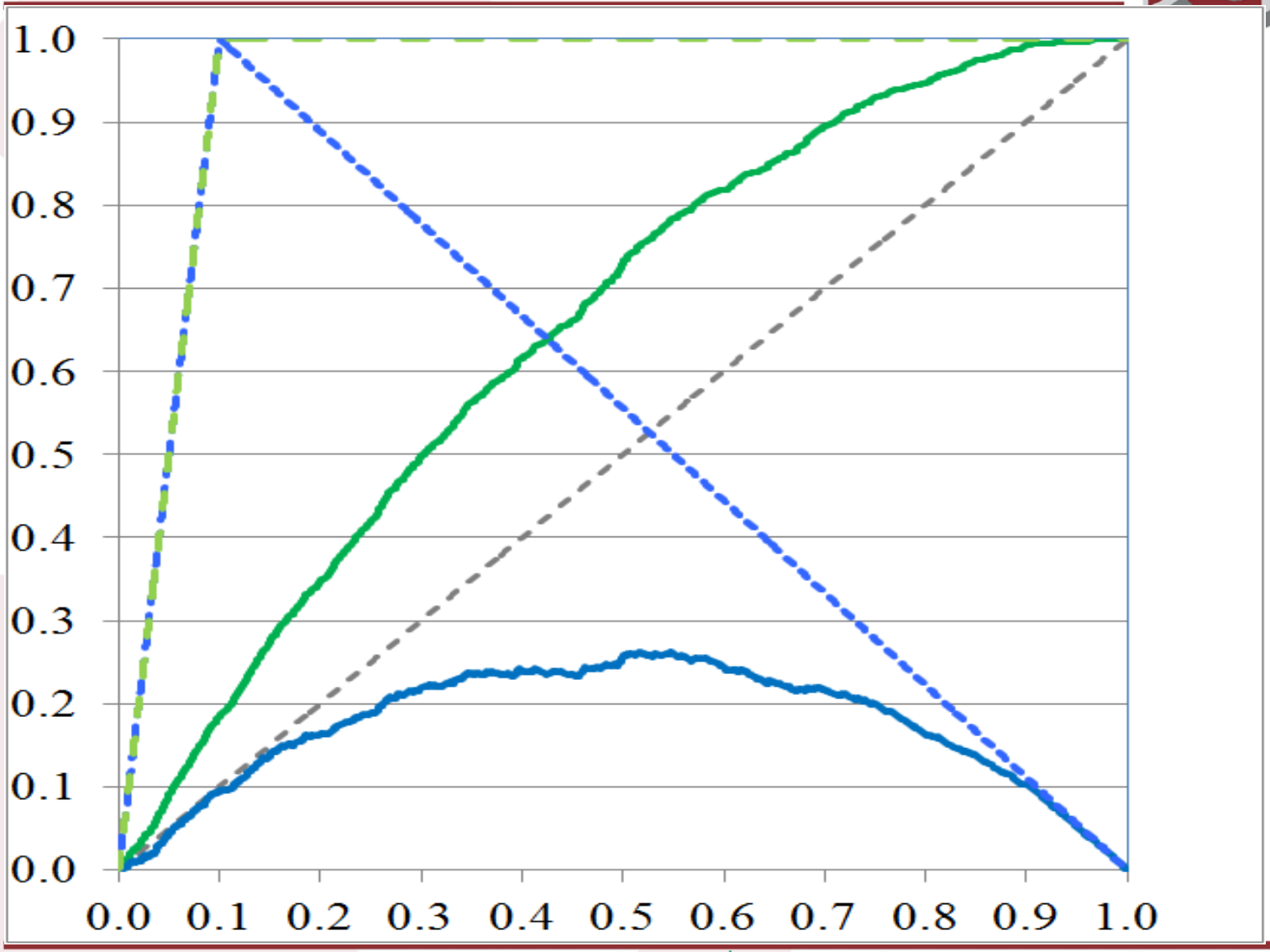
KS x Lorenz %Alvo=50%



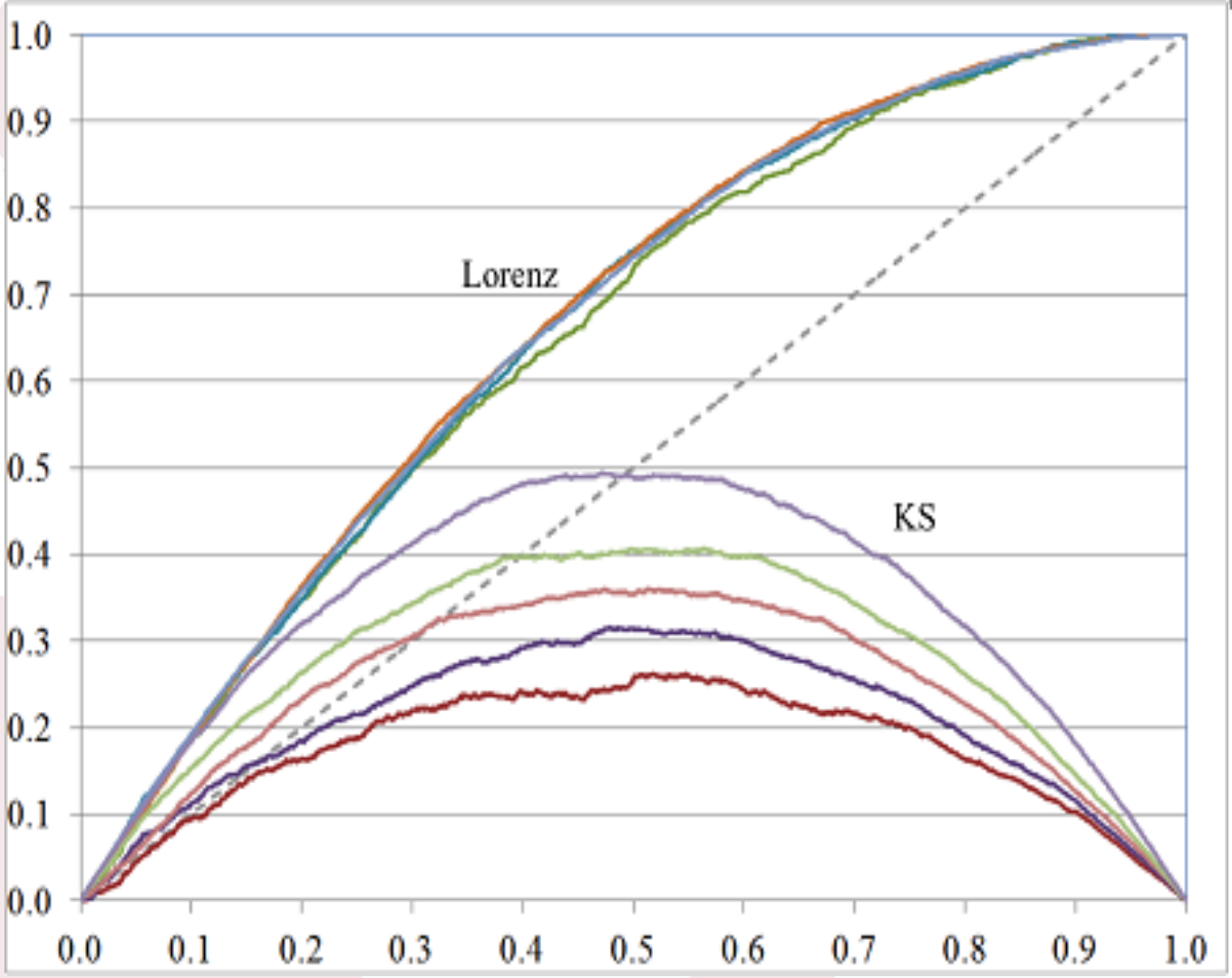
KS x Lorenz %Alvo=30%



KS x Lorenz %Alvo=10%



KS x Lorenz %Alvo=10% a 50%



Comparação de Métricas

% Target class	10%	20%	30%	40%	50%
Max_KS	0.262	0.316	0.360	0.406	0.494
AUC_KS	0.171	0.203	0.238	0.274	0.331
AUC_KS_Ratio	**0.341**	**0.405**	**0.477**	**0.548**	**0.662**
Gini_Area	0.154	0.162	0.167	0.164	0.165
Gini_Index	*0.307*	*0.324*	*0.334*	*0.329*	*0.331*
Gini_Index_Ratio	**0.341**	**0.405**	**0.477**	**0.548**	**0.662**

Comparação de Métricas

Feature	AUC ROC	AUC KS	Gini Index	AUC KS_Ratio
Value Range	[0.5;1.0]	[0.0;0.5]	Depends on the Prior [0.0;1.0)	[0.0;1.0]
Function type	Param.	Non-param.	Non-param.	Non-param.
Average and Error calcul.	Threshold or Vertical averaging	Simple averaging + StdDev	Simple averaging + StdDev	Simple averaging + StdDev
Interval assessm.	Specificity, Sensitivity	Score ranges	Score ranges	Score ranges
Multigrain assessm.	Via FROC	Not yet analyzed	Not yet analyzed	Not yet analyzed
Cost handling	Yes	Not yet analyzed	Not yet analyzed	Not yet analyzed

Conclusões

1. This paper has demonstrated the equivalence Gini_index_ratio = AUC_KS_ratio,

2. It links the Gini index to AUC_ROC and AUC_KS giving the data scientist a common ground for performance assessment of binary classification with different perspectives and tools.

3. The paper has proposed a unified area metric equivalent in all three representations: ROC, KS and Lorenz.
   AUC_KS_Ratio [=Gini_Index_Ratio =2*(AUC_ROC–0.5)]

1. It integrates all the knowledge on
   1. Lorenz curve from 1912
   2. Kolmogorov-Smirnov distribution from 1933 [10]
   3. ROC analysis from 1943

   bringing a new perspective to the field for interpreting binary decisions.

Conclusões sobre AUC_KS_Ratio

1. is an area-based metric suitable to the whole score range,
2. is non parametric so that the decision control is based on thresholding the abscissa,
3. is simply twice the AUC_KS,
4. the AUC_KS is easy to compute by the trapezium integration method,
5. the AUC_KS_Ratio ranges from 0 (for a chance classifier) to 1 (for the perfect classifier) scale,
6. the AUC_KS yields a simple calculation of average curves, and
7. The error in ROC curves for k-fold cross-validation can be calculated via the linear transformation which is much simpler and precise calculation than the vertical or threshold averaging.

Centro de Informática U·F·P·E

Obrigado!

?

Paulo ADEODATO

pjla@cin.ufpe.br